RC/RL Transients and Passive Filter Design

Time-constant measurement, first- and second-order frequency response, and LTspice correlation

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INSTRUMENT	Digilent Analog Discovery 3
SOFTWARE	Digilent WaveForms, LTspice
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Contents

1 Abstract

Two linked laboratory projects were carried out on the same three passive components, a 100 Ω resistor, a 0.1 µF capacitor, and a 1 mH inductor. The first characterized the step response of RC and RL circuits and extracted their time constants from oscilloscope captures. The second built RC low-pass, RC high-pass, and series RLC band-pass filters, measured their frequency response from 1 kHz to 500 kHz with a network analyzer, and compared the result against analytical transfer functions and LTspice simulation.

The measured RC time constant was 9.573 µs against a nominal 10 µs (−4.3%), and the RL time constant was 8.969 µs (−10.3%). All three filters behaved as designed. The more useful finding is that a single correction, using the capacitance implied by the measured RC time constant, brings the predicted stopband magnitude of the high-pass and band-pass filters to within 0.01 dB of measurement. The band-pass filter resonated at approximately 16.1 kHz with a quality factor of approximately 1.0.

2 Introduction

Resistors, capacitors, and inductors are the building blocks of signal conditioning in every avionics, sensor, and power system. Two properties decide how such a network behaves. The time constant sets how fast it responds to a step, and the transfer function sets which frequencies it passes. The two are views of the same physics: a first-order network with time constant τ has a corner frequency $f_c = 1/(2\pi\tau)$. Measuring both on the same parts gives an internal check that neither measurement provides alone.

The work was organized in two parts:

- **Project 1: transient response.** Apply a voltage step to series RC and RL circuits, record the resistor voltage, and extract the time constant.
- **Project 2: frequency response.** Build first-order low-pass and high-pass filters and a second-order band-pass filter, sweep them from 1 kHz to 500 kHz, and compare magnitude and phase against theory and LTspice.

3 Equipment and Method

3.1 Components and instrumentation

Table 1. Components and test equipment.

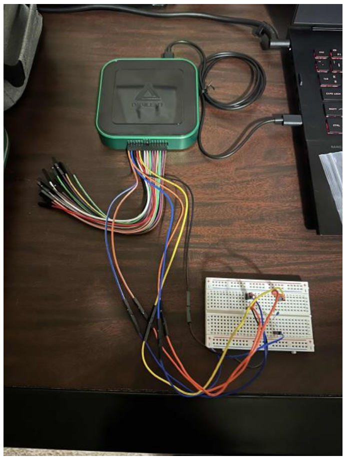
Item	Value or model
Resistor R	$100\,\Omega$
Capacitor C	$0.1\,\mu F$ ($100\,nF$), nominal
Inductor L	1 mH, nominal
Source and measurement	Digilent Analog Discovery 3 (waveform generator, 2-channel scope)
Analysis software	WaveForms v3.21.3 (Scope and Network Analyzer instruments)
Simulation	LTspice, AC analysis
Construction	Solderless breadboard, jumper leads

3.2 Transient measurement

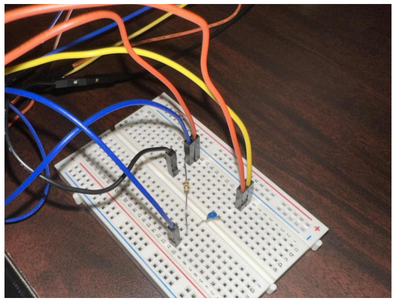
Channel 1 of the oscilloscope monitored the source and channel 2 monitored the resistor voltage. A 1 V square wave from the waveform generator supplied the step. On the rising edge, the first time cursor was placed at the start of the transition and the second where the resistor voltage reached its value at one time constant. The cursor difference is the measured time constant.

3.3 Frequency response measurement

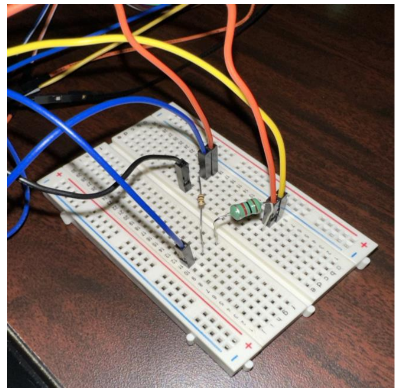
The Network Analyzer instrument swept a 1 V sine from 1 kHz to 500 kHz in 151 logarithmic steps, with channel 1 on the filter input and channel 2 on the output. It reports the ratio in decibels and the phase difference in degrees. Magnitude and phase values were read with on-screen cursors. Phase-crossing frequencies in Section 6 were digitized from the exported screenshots.



(a) Test setup.



(b) RC circuit.



(c) RL circuit.

Figure 1. Analog Discovery 3 connected to the breadboard, and the RC and RL transient circuits. Filter breadboards appear in Appendix A.1.

4 Project 1: Transient Response

4.1 Theory

For a series RC circuit driven by a step of amplitude V_s, the resistor voltage decays as the capacitor charges. For a series RL circuit, the resistor voltage rises as the inductor current builds:

$$V_R^{RC}(t) = V_s\, e^{-t/\tau}, \qquad \tau_{RC} = RC = (100)(0.1 \times 10^{-6}) = 10\ \mu\text{s}, \tag{1}$$

$$V_R^{RL}(t) = V_s \left(1 - e^{-t/\tau}\right), \qquad \tau_{RL} = L/R = (1 \times 10^{-3})/(100) = 10\ \mu\text{s}. \tag{2}$$

At $t = \tau$ the RC resistor voltage has fallen to $e^{-1} = 0.368\, V_s$ and the RL resistor voltage has risen to $1 - e^{-1} = 0.632\, V_s$. These are the thresholds used to place the second cursor. Both circuits were designed with the same 10 µs time constant so that the two measurements are directly comparable.

4.2 Results

Table 2. Measured and theoretical time constants.

Circuit	Threshold	τ theory [µs]	τ measured [µs]	Error	Implied f_c [kHz]
RC	$0.368\, V_s$	10.000	9.573	−4.3%	16.63
RL	$0.632\, V_s$	10.000	8.969	−10.3%	17.74

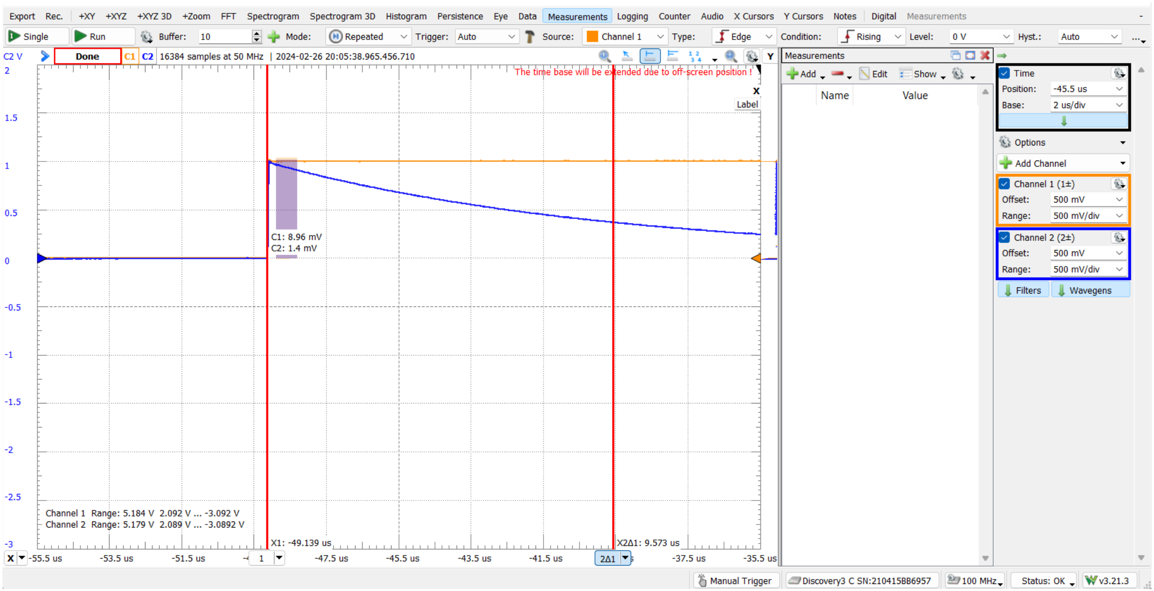


Figure 2. RC step response at 2 µs/div. The source (orange) steps to 1 V and the resistor voltage (blue) decays exponentially. The cursor difference gives $\tau = 9.573\,\mu s$. Zoomed captures of the same transient appear in Appendix A.2.

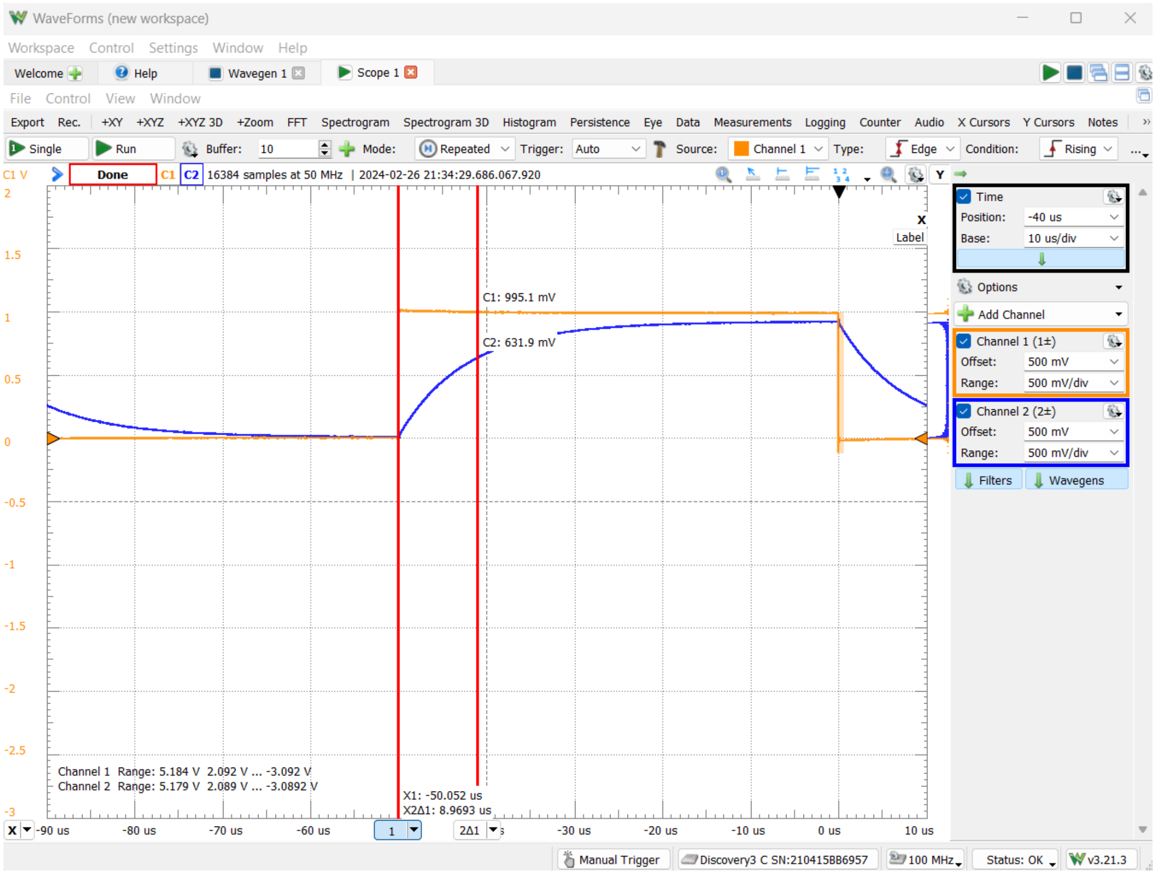


Figure 3. RL step response at 10 µs/div. The resistor voltage (blue) rises toward the source level. At the second cursor the readouts are 995.1 mV on the source and 631.9 mV on the resistor, giving $\tau = 8.969$ µs.

4.3 Cursor placement check

The RL capture records both channel voltages at the second cursor, so the threshold can be verified directly: $631.9/995.1 = 0.635$, against the target of 0.632. The cursor therefore sits very slightly past one time constant. Near $t = \tau$ the response rises at e^{-1}/τ per unit time, so the 0.003 overshoot corresponds to about 0.08 µs. The true RL time constant is closer to 8.89 µs, a correction smaller than the gap to theory and one that does not change any conclusion.

4.4 Discussion

Both measured time constants fall below the nominal 10 µs, but for different reasons.

RC circuit. The 4.3% shortfall is within the tolerance of a typical ceramic capacitor (commonly $\pm 10\%$). Solving $\tau = RC$ with R at nominal gives an effective capacitance of 95.7 nF. Section 6 shows that this value predicts the filter measurements independently, which is strong evidence that the capacitor, not the measurement, is responsible.

RL circuit. The 10.3% shortfall is larger and has a clear physical source. A real inductor has winding resistance in series with its inductance, so the circuit time constant is $\tau = L/(R + R_{\text{series}})$, not L/R. With L at nominal, the measured value implies a total series resistance of

$$R_{\text{total}} = \frac{L}{\tau} = \frac{1 \times 10^{-3}}{8.969 \times 10^{-6}} = 111.5\ \Omega, \tag{3}$$

or about 11.5 Ω beyond the resistor. That is a plausible winding resistance for a small 1 mH through-hole inductor. Breadboard contact resistance and inductor tolerance also shorten τ and cannot be separated with this data. Measuring the inductor's DC resistance with a multimeter would settle the split.

5 Project 2: Filter Design and Frequency Response

5.1 First-order filters

The low-pass filter takes its output across the capacitor and the high-pass filter takes it across the resistor. Both share the same corner frequency:

$$H_{\mathrm{LP}}(j\omega) = \frac{1}{1 + j\omega RC}, \qquad H_{\mathrm{HP}}(j\omega) = \frac{j\omega RC}{1 + j\omega RC}, \qquad f_c = \frac{1}{2\pi RC} = 15.92\ \text{kHz}. \tag{4}$$

At f_c both filters are 3 dB down. The low-pass phase is $-45°$ and the high-pass phase is $+45°$. Well beyond the corner, each rolls off at 20 dB/decade.

5.2 Second-order band-pass filter

The band-pass filter is a series RLC loop with the output across the resistor:

$$H_{\mathrm{BP}}(j\omega) = \frac{R}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}. \tag{5}$$

The reactances cancel at resonance, where the gain is unity and the phase is zero:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10^{-3})(10^{-7})}} = 15{,}915\ \text{Hz}. \tag{6}$$

Bandwidth and quality factor. For a series RLC circuit the half-power bandwidth is R/L in *radians per second*. Converting to hertz and forming the quality factor gives

$$B = \frac{R}{2\pi L} = \frac{100}{2\pi(10^{-3})} = 15{,}915\ \text{Hz}, \qquad Q = \frac{f_0}{B} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{10^{-3}}{10^{-7}}} = 1.00. \tag{7}$$

Note that R/L alone is the bandwidth in rad/s, not in hertz; the factor of 2π matters. Because $f_0 = B$ here, the shortcut $f_{c1,2} = f_0 \mp B/2$ would give 7.96 and 23.87 kHz. That shortcut assumes the passband is symmetric about f_0, which holds only for high Q. The exact half-power frequencies are

$$f_{c1,2} = f_0\left[\sqrt{1 + \frac{1}{4Q^2}} \mp \frac{1}{2Q}\right] = 9.84\ \text{kHz},\ 25.75\ \text{kHz}. \tag{8}$$

Their geometric mean is f_0, as it must be for a second-order band-pass response.

Table 3. Design values for all three filters.

Filter	Components	Output node	Key frequency	Additional
RC low-pass	R, C	across C	$f_c = 15.92\,\text{kHz}$	$-20\,\text{dB/dec}$ above f_c
RC high-pass	R, C	across R	$f_c = 15.92\,\text{kHz}$	$+20\,\text{dB/dec}$ below f_c
RLC band-pass	R, L, C in series	across R	$f_0 = 15.92\,\text{kHz}$	$Q = 1.00$, $B = 15.92\,\text{kHz}$

5.3 Measured transfer curves

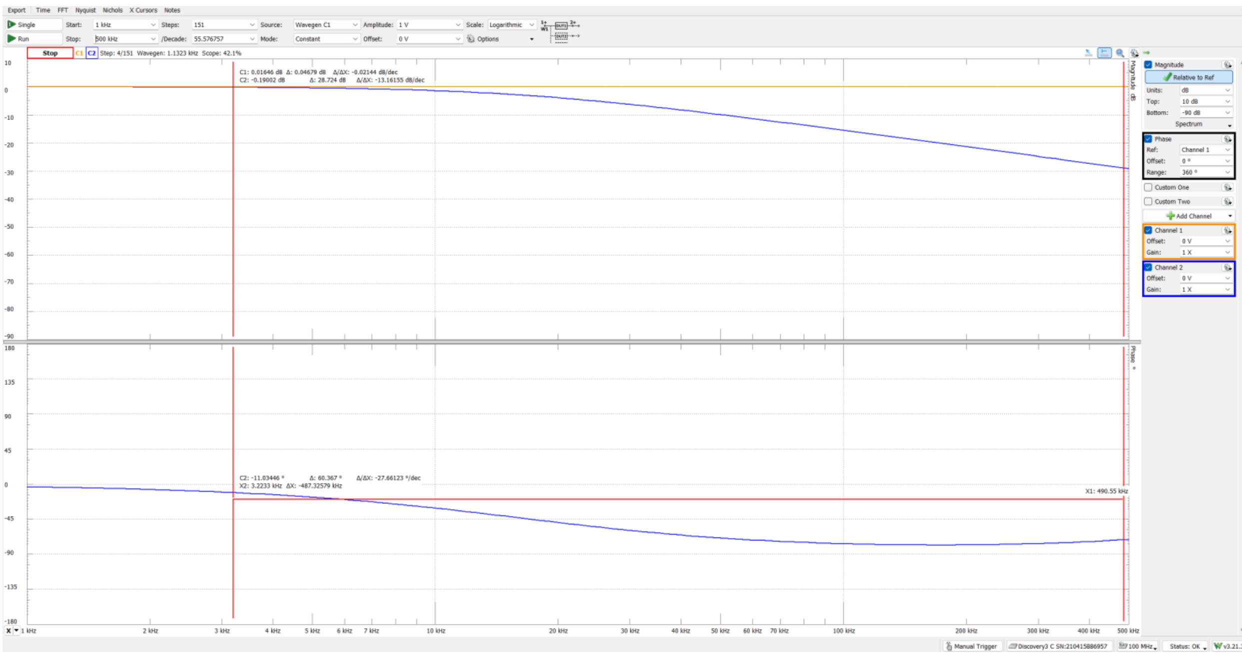


Figure 4. RC low-pass filter, measured. The magnitude (top) is flat through the low kilohertz range and rolls off above the corner. The phase (bottom) moves from 0° toward −90°. Cursor readings at 3.223 kHz are −0.190 dB and −11.03°. The second cursor at 490.55 kHz sits 28.72 dB and 60.37° below the first.

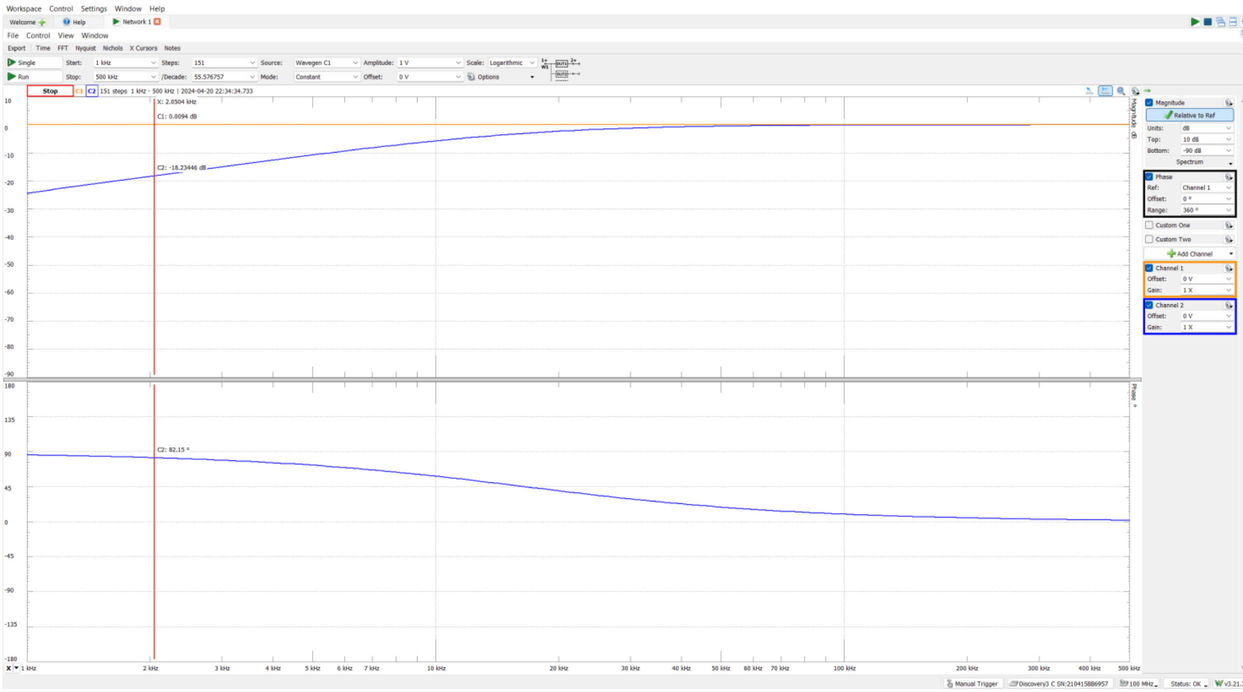


Figure 5. RC high-pass filter, measured. The response is the complement of Figure 4. The output is −18.23 dB at 2.050 kHz, rises through the corner, and flattens toward 0 dB. The phase falls from +90° toward 0° and reads 82.15° at the cursor.

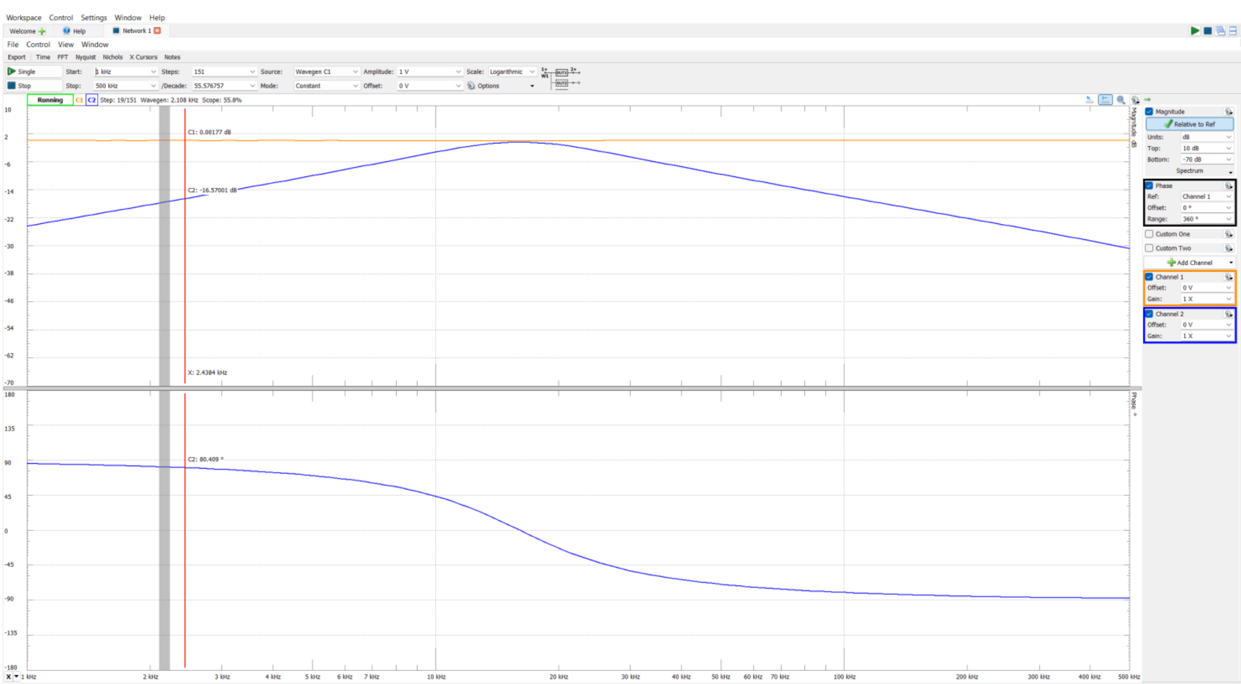


Figure 6. Series RLC band-pass filter, measured. The magnitude peaks near 16 kHz, close to 0 dB, and falls off on both sides. The phase passes through zero at resonance. The cursor at 2.438 kHz reads −16.57 dB and 80.41°. The wide, rounded peak is the signature of $Q \approx 1$.

5.4 LTspice simulation

Each filter was simulated in LTspice with an AC analysis from 1 kHz to 500 kHz. The simulation directive, visible on the band-pass schematic, is `.ac dec 1 1k 500k`. That setting evaluates the circuit at only 1 kHz, 10 kHz, 100 kHz, and 500 kHz, and LTspice joins those points with straight lines. The visible kinks in all three simulated traces fall exactly at the decade frequencies. This matters most for the band-pass filter: the highest simulated point is −2.85 dB at 10 kHz, so the plot never shows the 0 dB peak at 15.9 kHz.

The simulated values at the four sample frequencies do match the analytical transfer functions. Table 4 lists the analytical values. The LTspice traces pass through each of them to within the plot's reading resolution of about 0.5 dB.

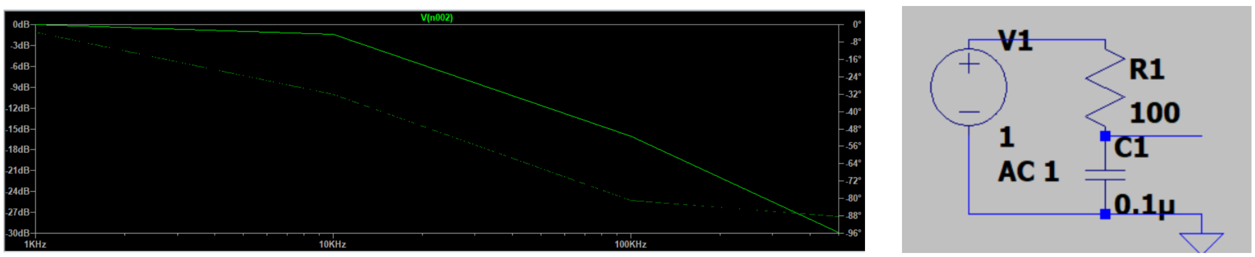
Table 4. Analytical magnitude and phase at the four frequencies LTspice evaluated.

	Low-pass		High-pass		Band-pass	
f [kHz]	dB	deg	dB	deg	dB	deg
1	−0.02	−3.6	−24.05	86.4	−24.02	86.4
10	−1.45	−32.1	−5.48	57.9	−2.85	43.9
100	−16.07	−81.0	−0.11	9.0	−15.86	−80.7
500	−29.95	−88.2	0.00	1.8	−29.94	−88.2

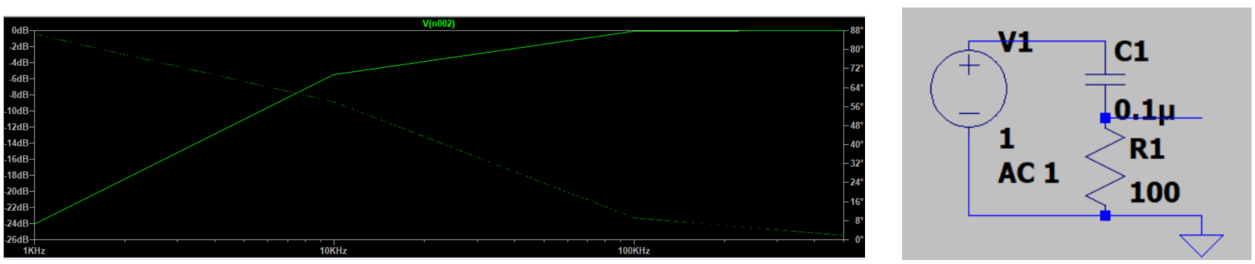
6 Comparison of Measurement, Theory, and Simulation

6.1 Magnitude and phase overview

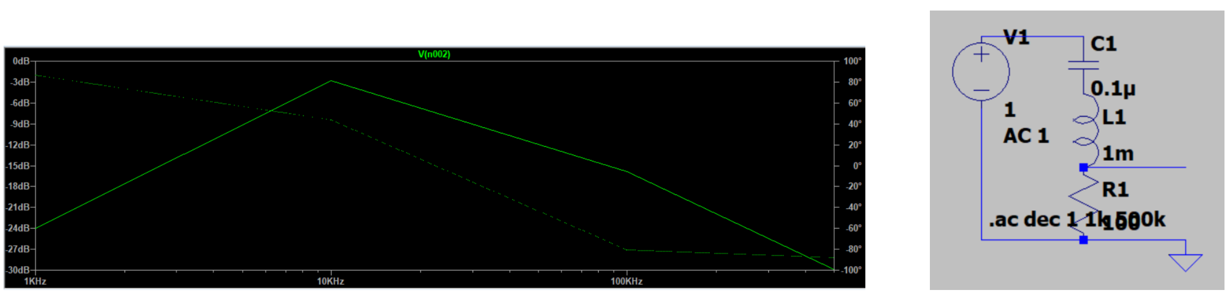
Figure 8 overlays the measured data on the nominal analytical response. Magnitude is shown at the recorded cursor points and phase as the full digitized trace. All three filters follow their predicted shapes over the full 1–500 kHz range. The measured phase curves are all shifted slightly toward higher frequency, consistent with the low capacitance identified in Section 6.2.



(a) Low-pass filter



(b) High-pass filter



(c) Band-pass filter

Figure 7. LTspice AC results (left) and schematics (right). Solid traces are magnitude and dashed traces are phase. The straight segments between decades come from the one-point-per-decade sweep, not from the circuit. The band-pass schematic shows the `.ac dec 1 1k 500k` directive.

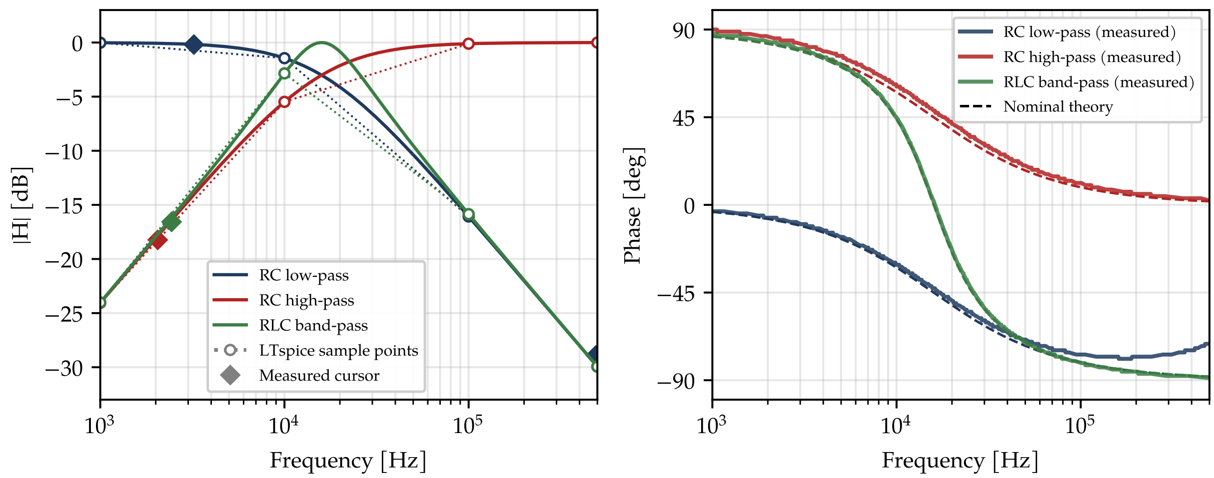


Figure 8. (Left) Nominal analytical magnitude for all three filters, with LTspice sample points (open circles) and measured cursor readings (diamonds). (Right) Measured phase digitized from the network analyzer exports (solid) against nominal theory (dashed).

6.2 One correction explains the stopband readings

The RC transient gave an effective capacitance of 95.7 nF (Section 4.4). If that value is correct, it should improve the filter predictions without any further adjustment. Table 5 tests this.

Table 5. Cursor readings against theory using nominal $C = 100$ nF and the effective $C = 95.7$ nF from the RC transient. L and R are nominal in both columns.

		Measured		Nominal C		Effective C	
Filter	f **[kHz]**	**dB**	**deg**	**dB**	**deg**	**dB**	**deg**
Low-pass	3.223	−0.19	−11.03	−0.17	−11.45	−0.16	−10.97
High-pass	2.050	−18.23	82.15	−17.87	82.66	**−18.24**	82.97
Band-pass	2.438	−16.57	80.41	−16.19	81.08	**−16.57**	81.47
Low-pass	490.55	−28.91	−71.40	−29.78	−88.14	−29.40	−88.06

In the stopband cursors, where the output is most sensitive to capacitance, the nominal prediction misses by 0.36 to 0.38 dB. The effective capacitance removes that error to within 0.01 dB in both filters, even though the value was taken from a separate experiment on a different day and the band-pass result also involves the inductor. At 3.2 kHz the low-pass filter sits almost at unity gain and is insensitive to C, but the effective value still improves the phase from a 0.42° error to 0.06°.

Two independent experiments agreeing this closely indicates that the as-built capacitor is about 4% below its marking, and that the measurement chain is sound.

6.3 Band-pass resonance and bandwidth

Digitizing the band-pass phase trace gives three crossings that locate the passband directly.

Table 6. Band-pass characteristic frequencies. Measured values are phase crossings digitized from the exported trace, with a reading resolution of about ±3%.

Quantity	Phase	Theory (nominal)	Measured
Lower half-power f_{c1}	$+45°$	9.84 kHz	9.93 kHz
Resonance f_0	$0°$	15.92 kHz	16.12 kHz
Upper half-power f_{c2}	$-45°$	25.75 kHz	26.32 kHz
Bandwidth B	–	15.92 kHz	16.39 kHz
Quality factor Q	–	1.00	0.98

The measured quality factor of 0.98 agrees with the design value of 1.00. The measured resonance is 1.3% above nominal, in the same direction as the low capacitance found above. The geometric mean of the two measured half-power points, $\sqrt{9.93 \times 26.32} = 16.17$ kHz, agrees with the measured zero crossing, which is an internal check on the digitization.

If the full 11.5 Ω implied by the RL transient were winding resistance, the band-pass peak would sit at −0.95 dB and the bandwidth would widen to 17.7 kHz. The measured bandwidth of 16.4 kHz lies between that value and nominal. This suggests the inductor's series resistance is real but smaller than 11.5 Ω, with some of the RL time-constant shortfall coming from inductor tolerance instead.

6.4 Behavior above 200 kHz

The low-pass phase does not settle at −90°. It bottoms out near −79° around 170 kHz and then climbs back to −71.4° at 490 kHz, and the magnitude there is 0.5 dB above the effective-C prediction. Both effects are consistent with a small resistance in series with the capacitor from equivalent series resistance, lead resistance, and breadboard contacts. That resistance adds a zero to the response. Adding 1.0 Ω in series with the capacitor gives −29.13 dB and −71.6° at 490.55 kHz, within 0.2 dB and 0.2° of the measurement. Parasitic resistance of this size is invisible at low frequency and dominant once the capacitor's reactance falls to a few ohms. It sets a practical floor of roughly −30 dB on the attenuation this breadboard filter can deliver.

7 Limitations

1. **Component values were not measured.** R, L, and C were taken at nominal markings. The effective capacitance and implied series resistance in this report are inferred from the data, not measured with an LCR meter.

2. **Single capture per measurement.** Each time constant comes from one cursor placement on one capture. Repeatability and cursor-placement uncertainty were not quantified beyond the threshold check in Section 4.3.

3. **Coarse simulation sweep.** One point per decade cannot resolve the band-pass peak or the shape of any curve between decades.

4. **Digitized phase data.** Phase-crossing frequencies were extracted from screenshots rather than

exported data files, which limits their resolution to roughly ±3%.

5. **Source and probe loading.** Generator output impedance and scope input capacitance were not included in any model. They matter most at the top of the sweep.

6. **Parasitics.** Capacitor ESR, inductor winding resistance and self-capacitance, and breadboard stray capacitance were not modeled explicitly. Section 6.4 shows that at least one of these is measurable above 200 kHz.

8 Conclusions

Transient response. The RC and RL circuits produced time constants of 9.573 µs and 8.969 µs against a nominal 10 µs. The RC shortfall of 4.3% is within capacitor tolerance. The RL shortfall of 10.3% is larger and consistent with inductor winding resistance lengthening the effective loop resistance to about 111.5 Ω.

Filters. All three filters behaved as designed. The low-pass and high-pass filters share a nominal corner of 15.92 kHz and showed complementary magnitude and phase. The band-pass filter resonated at 16.1 kHz with a measured quality factor of 0.98. That confirms the design value of $Q = 1.00$ and makes the filter broadband rather than narrowband.

Cross-validation. The most useful result links the two projects. The capacitance implied by the RC time constant, applied without any further fitting, brings the predicted stopband magnitude of both the high-pass and band-pass filters to within 0.01 dB of measurement. Two different instruments, measuring two different physical quantities on different days, agree on the same component value. That agreement confirms the measurement method more convincingly than either experiment alone.

Simulation. The LTspice models were correct, but their one-point-per-decade sweep hid the band-pass peak. Simulation resolution needs to be chosen for the feature being studied, not left at a setting that happens to draw a plausible curve.

8.1 Recommendations for future iterations

The most valuable change costs nothing. Rerun the LTspice analysis with at least 100 points per decade (`.ac dec 100 1k 500k`) and overlay the exported network analyzer data on the same axes. This turns the visual comparison into a quantitative one and resolves the band-pass peak.

Next, measure R, L, C, and the inductor's DC resistance with an LCR meter or multimeter before building. That would confirm or refute the inferred 95.7 nF capacitance and settle how much of the RL shortfall is winding resistance and how much is inductance tolerance.

Finally, export raw CSV data from WaveForms rather than screenshots, and repeat each transient capture several times. That provides a real uncertainty on each time constant and removes the digitization step from the phase analysis.

A Supplementary Figures

A.1 Filter breadboards

Figure 9. RC low-pass filter on the breadboard.

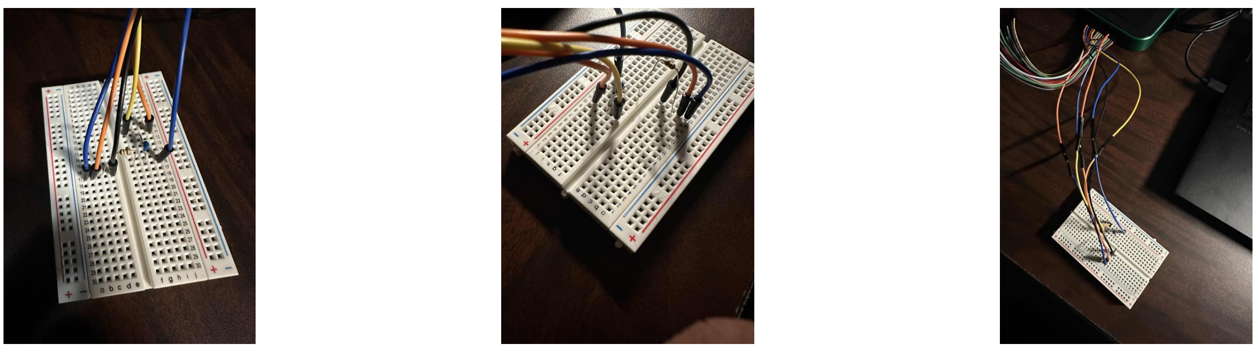


Figure 10. RC high-pass filter on the breadboard, with the capacitor and resistor positions swapped relative to the low-pass build.

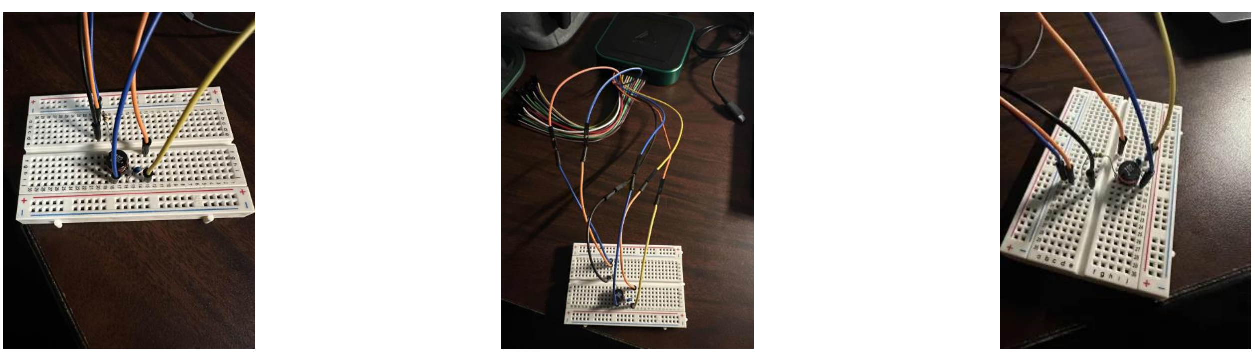
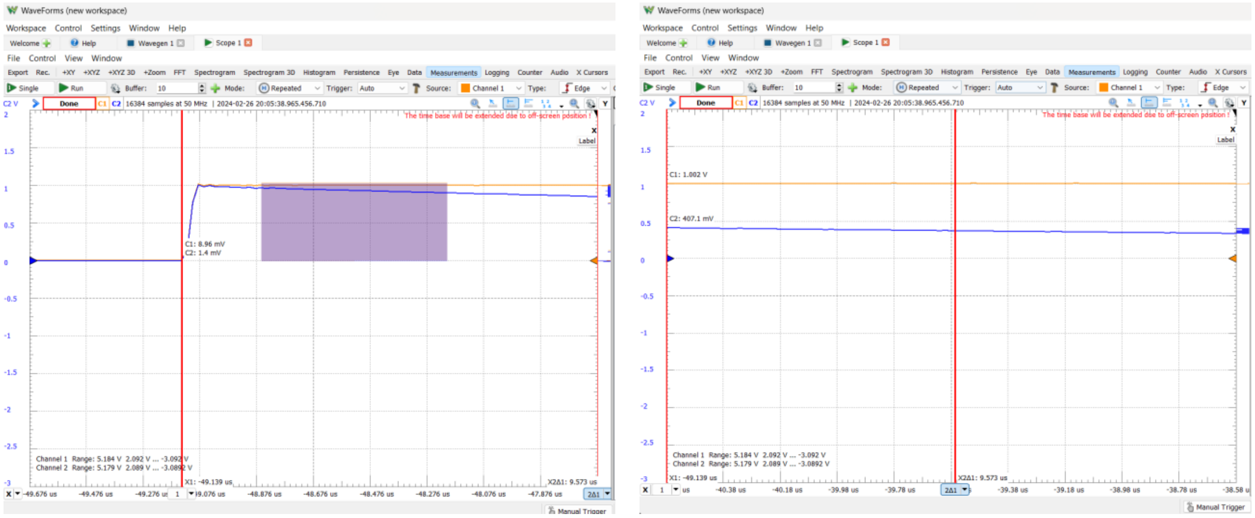


Figure 11. Series RLC band-pass filter on the breadboard.

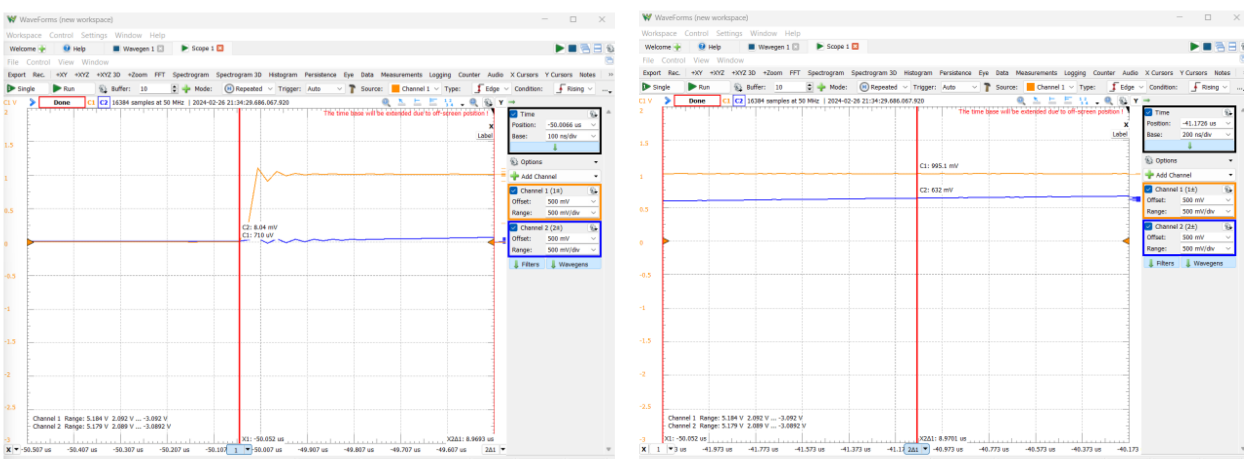
A.2 Additional transient captures



(a) RC, zoom on leading edge.

(b) RC, zoom near second cursor.

Figure 12. Zoomed RC captures from the same acquisition as Figure 2.



(a) RL, zoom on leading edge.

(b) RL, zoom near second cursor.

Figure 13. Zoomed RL captures from the same acquisition as Figure 3.

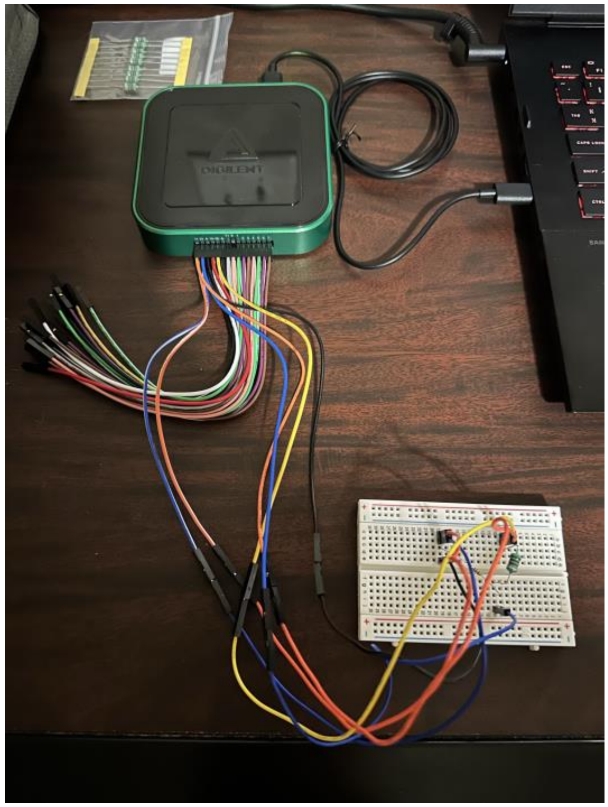


Figure 14. Instrument connection for the RL transient test.